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ABSTRACT

The Mathematics Pentathlon is a tournament of mathematics games held each spring since 1979 to promote the development of mathematical concepts and skills while fostering interaction among the educational community. There are five games in each of four divisions, for grades K-1, 2-3, 4-5, and 6-7. The focus is on active problem solving and critical thinking skills for children at different ability levels. This manual for K-1 first discusses problem solving, educational community involvement, the range of academic performance, and the use of physical models. Then each game is described, with mathematical content, materials, time, goals, starting procedures, game rules, and tournament rules stated, plus a reproduction of the gameboard. The games for this level are Kings and Quadraphages, Kalah, Hex!, Shape-Up, and Star Track. (MNS)

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# MATHEMATICS PENTATHLON™

A MANUAL OF
DIRECTIONS
AND
OFFICIAL TOURNAMENT RULES

DIVISION I
Grades K-1

Developed by:
John C. del Regato
Mary E. Gilfeather
Jane A. Schwarm

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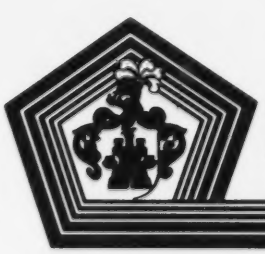

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# DEDICATION

Photo by Jimmy Mack

**David**

**Kelly**

**Tom**

**April**

***The only ladder to the stars is woven with dreams. It is never easy to keep reaching for dreams. Strength and courage can sometimes be lonely friends — but those who reach walk in stardust.***

**—Flavia Weedn**

# ACKNOWLEDGMENTS

The authors would like to express their indebtedness to the thousands of students, parents, teachers, and administrators who since 1979 have shared their dream and helped them make it a reality. Each new center for the Mathematics Pentathlon™ has attracted hundreds of dedicated educators as volunteers for this exciting mathlete tournament.

For their dedication and for their leadership in directing past Mathematics Pentathlon™ centers the authors are indebted to: Carey Chappell, Emma Jean Couture, Jane Morton, Donna Nunn, John Peterson, and Stephanie Warren. The Pentathlon could not have succeeded these past years without their guidance and most generous assistance.

It is with great pleasure that we thank the many past directors of Division I for their warmhearted cooperation and assistance in refining the game rules for tournament competition. They also gave generously of their time to help orient thousands of volunteers to their responsibilities as tournament officials. We gratefully acknowledge Kathy Coulliard, Sarah Goodenow, Sr. Paul Marie Gutgsell, Shannon Hudson, Augusta Jones McAnulty, Kayla Spector, Patricia Williams, and Dena Wirt.

For their generous financial contributions to the development of the Pentathlon, we express our thanks to Lilly Endowment and Wickes Foundation. Without their assistance we would not have been able to develop the tournament as rapidly as we have.

From the thousands of volunteers who either gave of their time or personally supported the authors in making the tournament grow, we gratefully acknowledge the contributions of: Bea D'Ambrosia, Darlene Atkins, Pam and Roger Barry, Sherry Bash, Ellen and Mark Bell, Mary Chappell, Tom Clark, Richard Crowe, Diane del Regato, Juan del Regato, Katie Doan, Betty and James Finkbeiner, Kathleen Fleming, Jo Ann Gentry, Paul Hamlin, Sally Hanagan, Marc Keisling, Paul Lindquist, Mike Metzger, Paula and Jim Minton, Vickie Moody, Robert O'Ana, Diane Renihan, Mike Ropp, Heinz Schwarm, Wayne Taylor, Ben Wilson, Mary Jo Wright, and Susan Zimmerman.

Lastly, we congratulate the mathletes who enthusiastically responded to the tournament and were placed in the Hall of Fame for their outstanding achievement.

March 15, 1986

John C. del Regato
Mary E. Gilfeather
Jane A. Schwarm

I wish to express my sincere appreciation to John del Regato and Mary Gilfeather for their excellent cooperation in preparing this manual. My sincere thanks also go to Marcia Kazmierzak, Sandy Barger, and Debbie Webb of the PDK staff for their assistance in the preparation of camera ready copy, and to Chuck Lawson and Paula Fleetwood in the printing of the materials. I owe a special debt of gratitude to Willard Duckett of the CEDR Center who acted as a consultant to members of my staff during my absence. Finally, special thanks to Wilmer Bugher, my predecessor, for not only identifying this program but for his excellent assistance, guidance, and support at all times.

This manual is part of the eighteenth program offered by Phi Delta Kappa's Center for Dissemination of Innovative Programs since the Center was organized in 1972.

March 15, 1986

Neville L. Robertson, Director
Dissemination Center
Phi Delta Kappa

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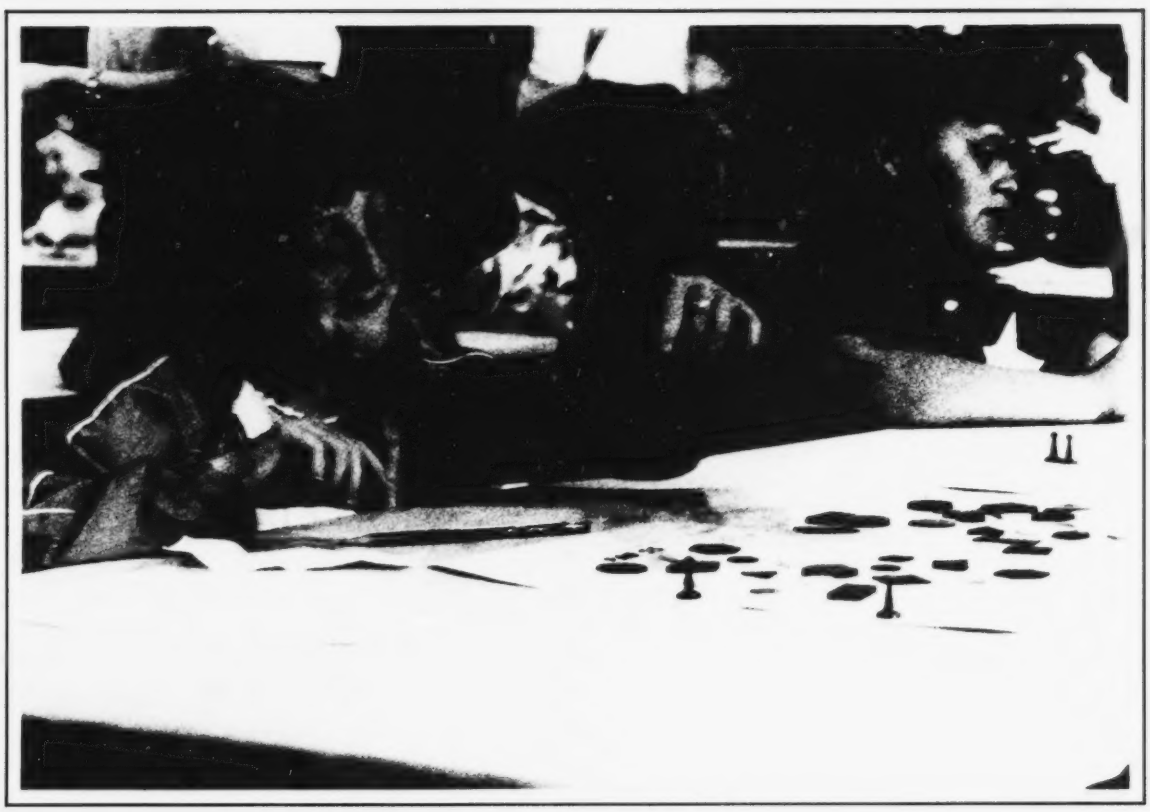

Photos by Eric Greulich

# INTRODUCTION

## The Mathematics Pentathlon™: Problem Solving in Action

The Mathematics Pentathlon™ brings parents, teachers, administrators, and learners together in an educational and social event which promotes the development of mathematical concepts and skills. This tournament of mathematics games held in spring appeals to a wide range of students at different ability levels and fosters interaction among all segments of the educational community. This event is referred to as a Pentathlon because there are 5 games in each of 4 divisions. Each division consists of two grade levels: Division I - grades K-1; Division II - grades 2-3; Division III - grades 4-5; and Division IV - grades 6-7. This event focuses on active problem solving and critical thinking skills. National assessments and international studies have shown that most children lack a true understanding of mathematics on a higher level which involves the development of critical thinking skills, logical and hypothetical reasoning, discerning patterns, and seeing new relationships. With this concern in mind, professional education organizations have often emphasized problem solving and reasoning skills. Continually mounting evidence suggests that the Pentathlon offers an excellent solution to this need.

The Pentathlon was organized as a tournament of mathematics games in 1979 as a tribute to the International Year of the Child. Since then it has become an annual event in the Tri-County area (Saginaw-Bay-Midland) of Michigan. This Center for the Mathematics Pentathlon is now under the leadership of parents and educators who believe it to be a most rewarding educational experience. In 1981 a new Tri-State Center (Indiana, Kentucky, Illinois) was established in Evansville, Indiana. Over the past years both of these centers have provided parents and educators with orientation sessions and in-service education programs related to the tournament. An ever-increasing volunteer support group of teachers, parents, administrators, and students has provided the best evaluative index of the impact this tournament has made.

In 1984 the Indianapolis center for the Mathematics Pentathlon™ was established. At the same time the National Headquarters for the Pentathlon was organized. Also located in Indianapolis, this headquarters is responsibile for assisting other centers and establishing new ones on a national basis. Since then a number of other centers for the Mathematics Pentathlon™ tournament have developed. It is estimated at this writing that well over 100,000 mathletes nationally are involved in practicing these problem-solving games for tournament competition.

What has caused this dramatic involvement in a problem-solving tournament? Why is the Mathematics Pentathlon™ growing exponentially in the numbers of individuals involved? The answers to these questions appear to lie in the unique way this tournament has approached the crisis currently faced in mathematics education. Some of the unique features of the Mathematics Pentathlon™ are: 1) its focus on active problem solving; 2) its involvement of the total educational community; 3) its acceptance of a broad range of academic performance levels by participants; 4) its use of concrete and pictorial manipulatives; and 5) its balanced use of non-verbal and spatial reasoning.

### Active Problem Solving

Alvin Toffler and other futurists have brought to our attention that our future is going to involve change at a rate even greater than present generations have experienced.

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This implies that future problems will involve a dynamic component which will be undergoing change even as those problems are being studied. It is clear then that a fundamental skill for dealing with the future is active problem solving, i.e., the ability to solve problems which are undergoing change during the process of attempts to resolve them.

The Mathematics Pentathlon™ provides experiences in thought processes necessary for active problem solving. Through judicious use of logical analysis, spatial reasoning, computational reasoning, strategic thinking, and chance, the Pentathlon mathlete attempts to resolve problems created by five entirely different two-person game events. Matched with a different opponent in each of the five tournament events, a mathlete attempts to select a best option from a cluster of continually changing options. The Pentathlon game events rely to a great extent on the use of strategies. This provides experience in inductive and deductive reasoning through the use of systematic implementation of sequential thought. Such thought patterns are directly related to real life problem-solving situations. For example, the idea of networking is a combination of several different simultaneous approaches to the same problem.

Unlike other contests and competitions which provide individuals with static routine problems for solution by applying known algorithms, the Pentathlon focuses on non-routine types of problem-solving situations that foster the development of active problem-solving skills. But this emphasis on a dynamic approach to solving mathematical problems is not the only characteristic which makes the Pentathlon a unique experience.

## Total Educational Community Involvement

The Pentathlon games were designed so that adults as well as children would enjoy them. Just as athletic programs are often supported by the home and school, the Pentathlon nurtures home and school involvement through supporting the efforts of each mathlete who comes to a tournament. The great interest by both adult and student may be traced in part to the dynamic problem-solving features mentioned above. Parents, teachers, and administrators have found these games to be of educational value because they cover a variety of mathematical concepts and skills such as: basic arithmetic operations, fractions, decimal arithmetic and fraction/decimal conversion, classification and logic, and tranformational geometry to name a few.

Currently there are four divisions within this tournament with each division encompassing two grade levels: Division I - grades K-1, Division II - grades 2-3, Division III - grades 4-5, and Division IV - grades 6-7. A new Division V for grades 8 and 9 is presently being constructed. Within each division there are five different games which reflect the mathematical concepts and skills appropriate for that grade level.


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During the months of August through December, parents and educators are oriented to the games that comprise the tournament. These individuals then teach their children the games at home and/or in school and prepare and select those mathletes who will register for tournament competition. These same individuals also decide whether or not they will volunteer their services as one of the Pentathlon tournament officials or as one of the many volunteer assistants. Another unique feature of the Mathematics Pentathlon™ is the ratio of mathletes to tournament volunteers — 2:1! This type of volunteer support is only possible because of the pride and ownership that parents and educators feel toward this event.

## Representatives From a Broad Range of Academic Performance

Currently the Pentathlon consists of a total of 20 different games. Each of these games is either an original creation or some significant modification of a known game. Considerable care has been taken in designing the five different two-person games for each division. Factors which entered into the development process included: 1) creating games sensitive to the developmental differences of students; 2) providing well defined rules and instructions for tournament play; 3) designing games which support and supplement formal mathematics curricula; 4) making the games appealing to both students and adults by blending both strategy and chance within the games; and 5) emphasizing the variety of ways manipulation of physical models can be used in problem solving. The first criterion in this list has enabled the Pentathlon to attract students from a broad range of academic performance. Since the games employ non-verbal as well as the more traditional symbolic and computational aspects, many students who are strong in non-verbal or spatial reasoning abilities are attracted to the Pentathlon games which require this type of thinking. Often a child may be very skilled in this type of reasoning and yet be either disinterested and/or poorly skilled in the verbal and quantitative skills so valued within formal instructional settings. Many such students have found the Pentathlon experience to be rewarding not only because it values spatial reasoning and non-verbal ability, but because it provides a vehicle for motivating the development of logical and quantitative thinking. The child of average academic performance may also be similarly motivated. And, the student who excels academically is also challenged by the games which require multiple classification and/or geometric and spatial reasoning. The active problem-solving feature discussed above fascinates all. But, the creative use of physical models to represent the underlying mathematical concepts is yet another reason why the Pentathlon has such a broad range of appeal.

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## Physical Models

The tournament games employ a variety of educational manipulatives. Examples of such materials are pattern and attribute blocks, Cuisenaire rods, pentominoes, dominoes, and fraction bars. Although research studies have indicated the great value these and other physical models have for developing children's mathematical thinking, children often have little or no experience with these valuable learning tools.

Problem solving most typically requires the solver to build a model or create a picture of the problem. Many students have not been provided with experiences which make use of physical or pictorial models in the development of mathematical ideas. Typically these students lack the necessary imaging for resolving mathematical problems that have a spatial reasoning component. Since all of the mathematical ideas studied in grades K through 12 can be communicated using visual models, it appears most reasonable to enrich a child's experience by the appropriate use of concrete and pictorial instructional manipulatives.

Parents and educators are often surprised by the way in which physical models enable a child to investigate mathematical relationships that were not easily seen with the use of symbols alone. Parents appreciate the value of these objects as they see their child conquer the strategies used in a Pentathlon game. These parents then become most supportive of the need and use of these materials in their child's ongoing formal instruction. Teachers and administrators also gain a renewed respect for the way in which physical and pictorial models enrich the imaging related to mathematics concept development.

## The Future

It is through the efforts of many dedicated parents and teachers that the Mathematics Pentathlon™ has become an established annual event. All indicators point to a bright future for this unique tournament. If you would like to see a Pentathlon, attend one of the many planned orientation sessions, or arrange your own inservice workshop, call (317) 926-MATH for further information. There is no doubt that the Pentathlon's focus on active problem solving is providing an avenue for children to experience tomorrow's mathematical skills. We can no longer look to the future through a rearview mirror. Rather than a nostalgic return to the past emphasis on the basics of arithmetic memorization and drill, the Pentathlon stresses fundamentals that are future oriented. By acquiring active problem-solving skills, mathletes can look to future challenges with confidence.

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# KINGS AND QUADRAPHAGES

**Created by**
**Dr. John C. del Regato**

Both players attempt to isolate the opponent's king in this two-person Mathematics Pentathlon™ strategy game. The quadraphages (square eaters) are used by each player to entrap the opposing player's king. Players receive practice in horizontal, vertical, and diagonal movement so basic to many concepts in mathematics and science. A dynamic use of counting skills becomes strategically important as each player's king becomes more and more entrapped.

**UNDERLYING MATHEMATICS/SCIENCE:** horizontal, vertical, and diagonal movement on a grid, counting, the topology of closed and open regions; observation skills, hypothesizing and experimentation; deductive and inductive reasoning

**MATERIALS:** Official Mathematics Pentathlon™ gameboard, two pawns of differing color, sixty chips — thirty each of the same two colors as the pawns.

**TIME:** Total tournament playing time is 15 minutes. Each player has one minute to complete a turn.

**GOAL:** To win, a player must succeed in entrapping the opponent's king so that it is unable to move.

**TO START:**

A. The two kings (pawns) are placed into a container so that their color cannot be seen. One color is declared the starting color. Players select their pawns. The player with the starting color begins the game.

B. The kings (pawns) are placed into the spaces marked with a crest.

C. Each player gathers 30 quadraphages (chips) which correspond to the color of the king.

**RULES:**

1. **Taking a Turn:** Players alternate turns. During a turn, a player first moves the king (pawn) to any adjoining unoccupied space (i.e. one space horizontally, vertically, or diagonally) and has a quadraphage "eat a square" by placing a chip on any remaining unoccupied space.
2. **Moving the King:** Once a player has moved the king to an adjoining space and once a player's hand is released from the pawn, it cannot be repositioned.
3. **A Piece Laid Is a Piece Played:** Once a player's hand is removed from a Quadraphage (chip) placed onto one

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of the gameboard's squares it may not be moved to another square.

4. **"Eaten Squares":** Squares covered by chips have been "eaten" by quadraphages and therefore can no longer be moved into by either king.
5. **The Crest Marked Box:** Players may move into the crest marked box during play; it is treated as any other square box. The crest mark has no significance other than indicating the starting positions for each player's king.
6. **Beginning a Turn:** A player's turn begins when an opponent's hand is removed from placing a Quadraphage (chip) onto the gameboard.
7. **End of Turn:** A player's turn ends when the player's hand is released from placing a Quadraphage (chip) onto the gameboard.
8. **Win/Tie:** Play continues until one of the kings is trapped and can no longer move horizontally, vertically, or diagonally into an adjoining square. A player whose king is trapped has lost the game and the opposing player is declared the winner.

**TOURNAMENT RULES:**

9. **Challenges:** If a player violates any of the preceding rules, the opposing player may issue a challenge after a player's hand is removed from placing a Quadraphage (chip) onto the gameboard. A Pentathlon official will rule on any challenge. If the challenge is ruled to be correct, the challenger can remove one of the opponent's chips from the board and place the opponent's king on any unoccupied space on the gameboard. If a challenge is ruled to be incorrect, the challenger must forfeit a turn. Challenges *may not* be issued after a player has touched the king.
10. **One Minute Time Limit:** Each player must complete a turn within one minute. If a player fails to complete a turn in one minute, the king (pawn) will be placed back into the position where it was at the beginning of that turn and the player will forfeit that turn. The third violation of this rule results in a forfeiture of the game. Players may not challenge each other over this rule at the Pentathlon. An official at the tournament will decide if this is being violated.
11. **Game Time Limit:** At the Pentathlon, if the fifteen (15) minutes of game time has run out before either play has ended the game, a tie will be declared. If a player is in the process of completing a turn when game time is called, that turn will be allowed to be completed. If that turn results in entrapping the opponent's king, a win will be declared.


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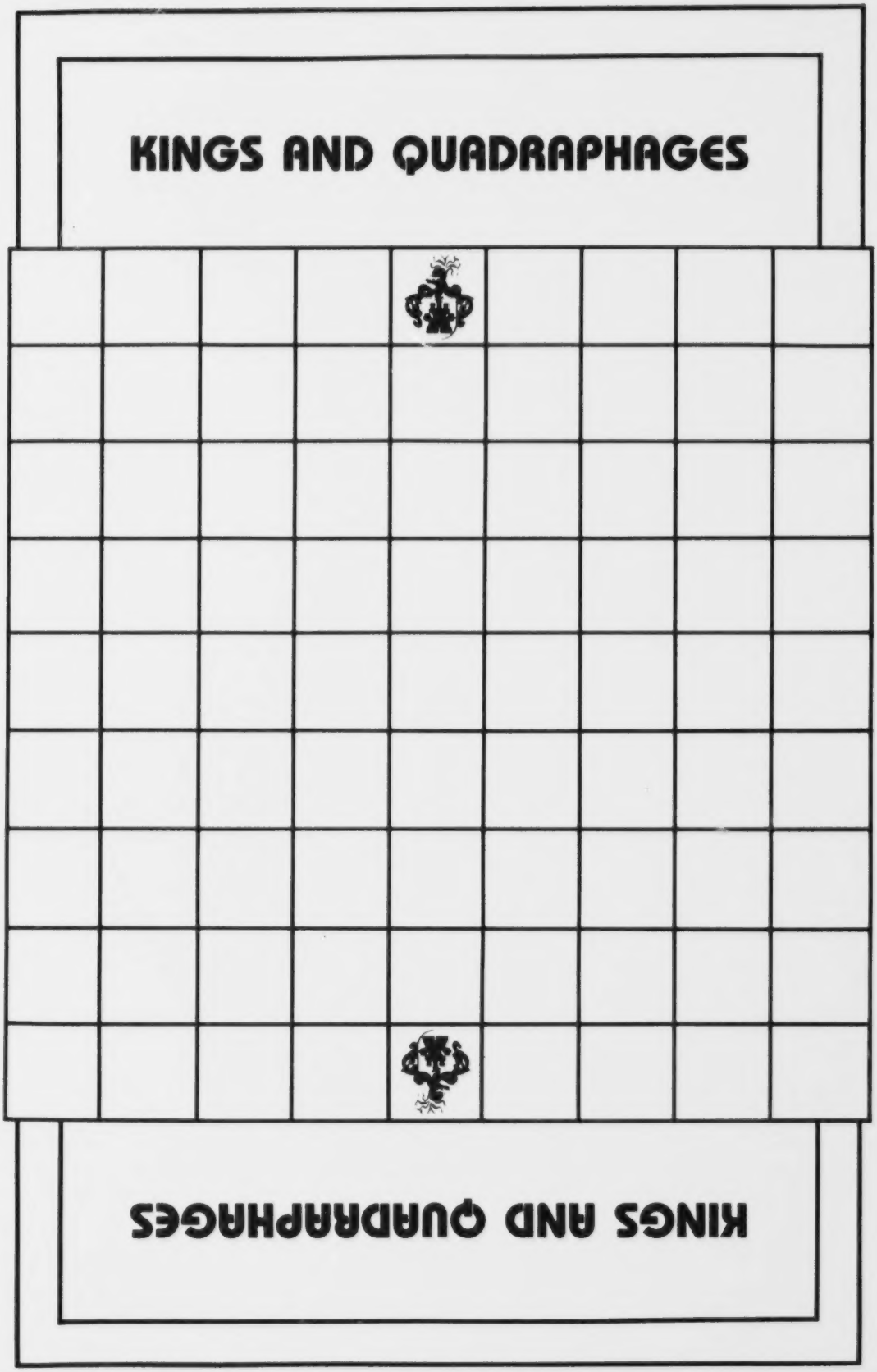
# KINGS AND QUADRAPHAGES



KINGS AND QUADRAPHAGES

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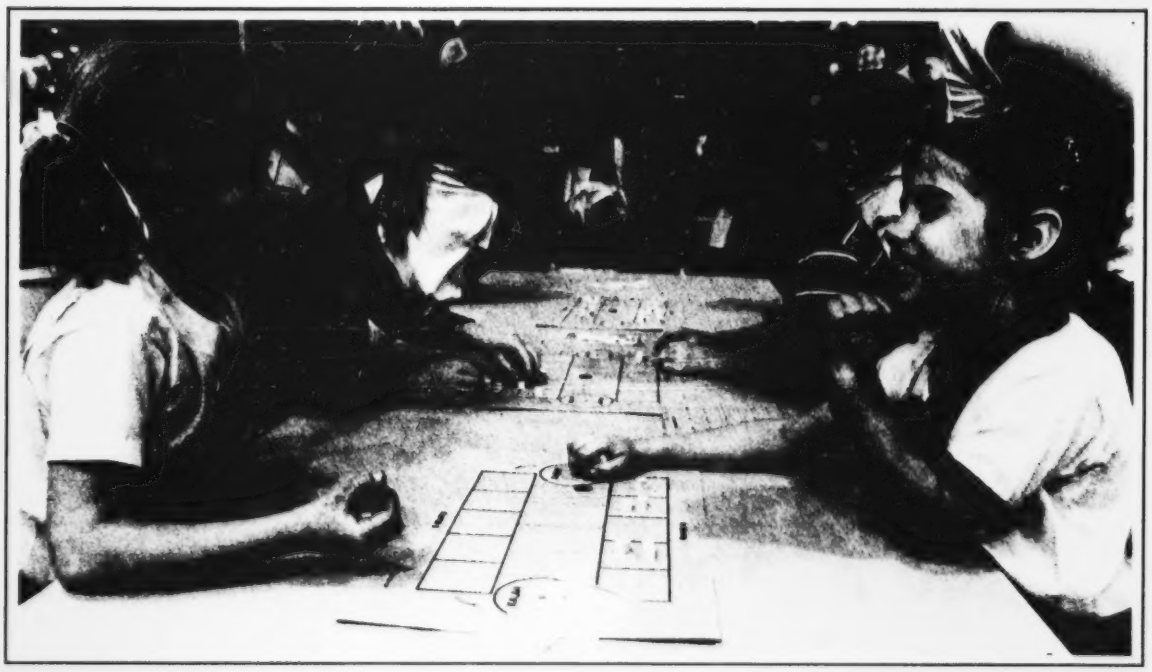



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# KALAH

**Created by**
**Dr. John C. del Regato**

Kalah is a version of one of the many mancala ancient African and Asian counting games. Mancala games are traditionally played on large boards carved of wood or hand dug pit holes carved by hand into the earth. Seeds, beans, sea shells, or small stones are generally used in the more ancient mancala games. The official Kalah version played at the Mathematics Pentathlon ™ makes use of a specially designed gameboard with "shields" and circular "kalahs" onto which centimeter cubes are positioned. Counting and quantitative judgment skills are practiced in this fascinating strategy game as players analyze the evolving structural relationships of the playing pieces (cubes) distributed among the "shields" on the gameboard.

**UNDERLYING MATHEMATICS/SCIENCE:** counting, addition, greater than — less than, equality, one-to-one correspondence; observational skills; deductive and inductive reasoning

**MATERIALS:** Official Mathematics Pentathlon™ gameboard, 32 centimeter cubes, and a coin

**TIME:** Total tournament playing time is 15 minutes. Each player has one minute to complete a turn.

**GOAL:** The kalah (circular space) containing the most cubes at the end of the game, designates which player has won. If both kalahs contain the same number of cubes at the end of a game, a tie is declared.

**TO START:**

A. A coin is tossed to determine the starting player. One player calls either heads or tails. If the call is correct, the player begins the game. If not, the other player begins.

B. At the start of the game, 4 cubes are placed into the two center shields on each player's side of the board. Three cubes are placed into each of the other eight shields.

C. Each player's kalah is the circular space on the player's right side. One circle is labeled H and the other S indicating to which player it belongs (see gameboard). There should be no cubes in either kalah at the game's start.

**RULES:**

1. **Alternating Turns:** Players alternate taking turns in trying to accumulate as many cubes as possible in their own kalah.

2. **Cube Selection and Placement:** Each player in turn picks up all of the cubes from any one of the player's own five shields and places them, one by one into adjoining con-

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secutive shields or player's own kalah. Cubes are placed one to a shield or kalah in a counterclockwise direction (see arrows on gameboard).

3. **Exception to Consecutive Placement:** No cubes are ever placed into an opponent's kalah. Should a player have additional cubes after placing one cube in each of the opponent's five shields, the opponent's kalah is bypassed. The remaining cubes are distributed beginning on the player's left side of the board as described in rule 2.
4. **Selected Cubes Must Be Played:** Once a player has removed any cubes from one of the five shields, that turn must be completed. After drawing out all cubes from a chosen shield, a player may not return the cubes to that shield and make a different selection.
5. **Additional Turn:** If the last cube to be placed on the gameboard lands in the player's kalah, an additional turn is granted. Many additional turns can be repeatedly achieved by the strategic placement of cubes.


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6. **Capture:** If the last cube to be placed on the gameboard lands in an empty shield on the player's side of the board, all of the opponent's cubes contained in the shield directly across the board from where the last cube was placed (see dashed connecting lines on gameboard) are captured and put into the player's kalah. No additional turn is granted.
7. **Capture Time Limit:** If a capture is not completed within 10 seconds (after placing the last cube in an empty shield on the player's side of the board), the opposing player may start a turn. In effect, the opportunity to capture an opponent's cubes has been lost.
8. **End of Turn:** A turn ends when the last cube to be placed onto the gameboard has been released by a player or, in case of a capture, when the captured cubes are placed into the player's kalah.
9. **Beginning of a Turn:** A player's turn begins either:
   a. at the signaled beginning of the game,

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b. at the end of the opponent's turn (see rule 8),

c. when an official has indicated play to continue after a challenge has been issued (see challenge rule below), or

d. 10 seconds after an opponent has failed to complete a capture.

10. **End of Game:** At the beginning of a turn or during an additional turn, if there are no cubes in any of the player's shields, the game has ended.

11. **Win/Tie:** Only those cubes which are contained in each kalah are compared at the end of the game. The winner is the player having the kalah which contains the most cubes. If both players have the same number of cubes in their kalahs, a tie is declared. If seventeen or more cubes are amassed in a player's kalah before the end of the game, an automatic win is declared.

**TOURNAMENT RULES:**

12. **Challenges:** If a player violates any of the preceding rules the opposing player may issue a challenge. An official will rule on this challenge. If the challenge is found to be correct, the gameboard must be restored to where the cubes were located at the beginning of the challenged player's turn and the challenged player loses a turn. If a challenge is found to be incorrect, the challenger forfeits a turn.

13. **One Minute Time Limit:** Each player must complete a turn within one minute or suffer a forfeiture of a turn. Three calls of this nature at the Pentathlon will result in a forfeiture of the game. Players may not challenge each other over this time rule at the Pentathlon. An official at the tournament will decide if this rule is being violated.

14. **Game Time Limit:** At the Pentathlon, if the fifteen (15) minutes of game time has run out before either player has ended the game, the winner will be declared to be the player whose kalah contains the most cubes at that time. If both players have the same number of cubes, a tie will be declared. If a player is in the process of completing a turn when game time is called, that turn will be allowed to be completed.



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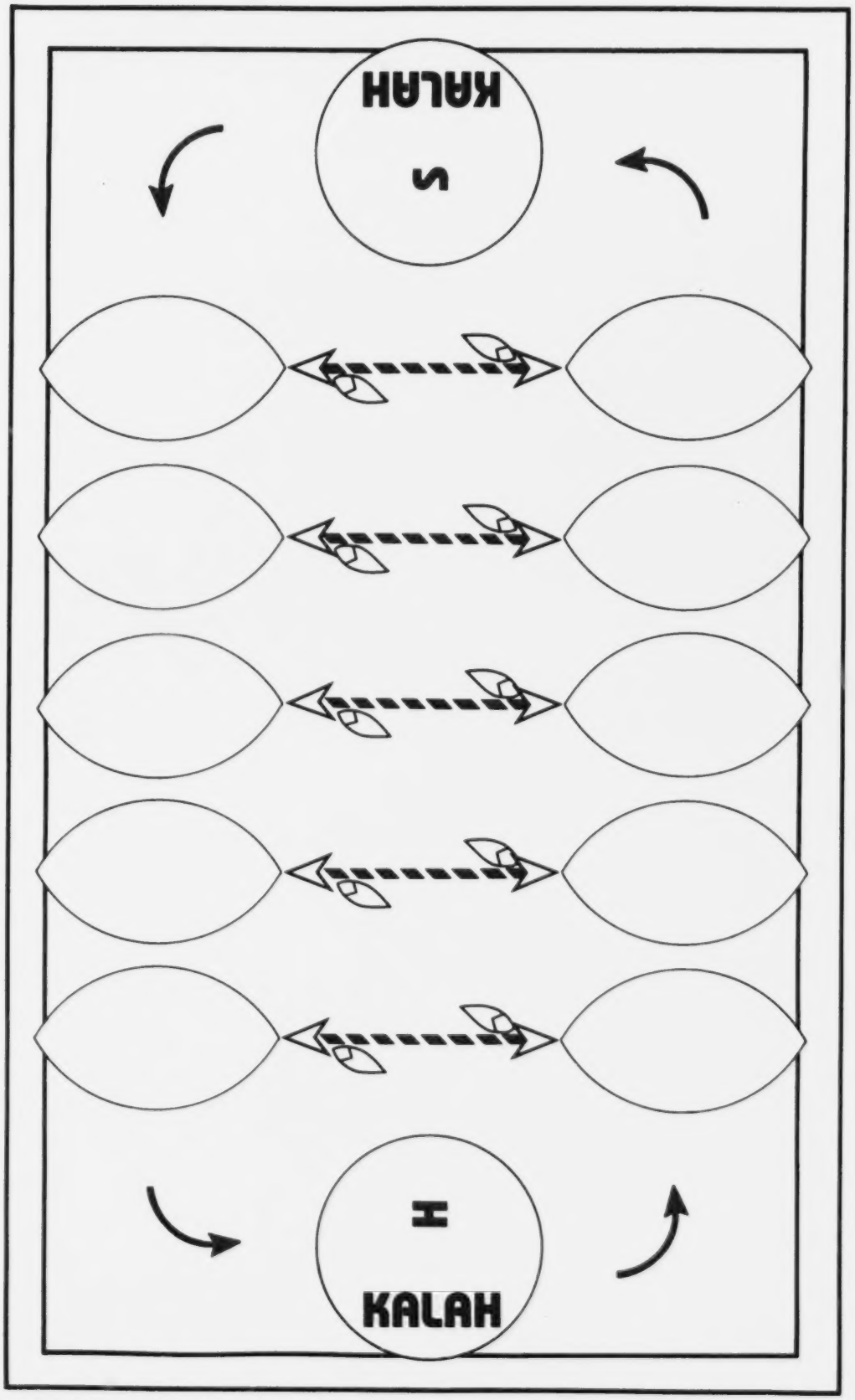


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# HEX!

**Created by**
**Dr. John C. del Regato**

The informal use of fractions in a concrete form highlights this Mathematics Pentathlon™ event as players alternate in placing Pattern Blocks onto designated geometric regions. Aspects of Number Theory were used to design this game. The relationships among the green triangles, blue rhombi, red trapezoids, and yellow hexagons of a Pattern Block set is important in order to uncover a winning strategy for this game. Many adults have found this game intriguing for the geometric twist that can alter the results of a strategy based strictly on the underlying mathematical structure. Often, attractive geometric patterns evolve as players attempt to be the last player to place a Pattern Block onto the gameboard.

**UNDERLYING MATHEMATICS/SCIENCE:** shape recognition, structural relationships of shapes, fractions, counting and addition; observation skills, hypothesizing and experimentation; deductive and inductive reasoning

**MATERIALS:** Official Mathematics Pentathlon™ gameboard, Pattern Blocks: six each of yellow hexagon, red trapezoid, blue rhombus, green triangle, orange square, tan rhombus

**TIME:** Total tournament playing time is 15 minutes. Each player has one minute to complete a turn.

**GOAL:** To win, a player must be the last one able to place one of the available Pattern Block pieces onto the gameboard. This *does not* imply that the gameboard will be completely covered with Pattern Blocks.

**TO START:**

A. A coin is tossed to determine the starting player. One player calls either heads or tails. If the call if correct, the player begins. If not, the other player begins. At the Pentathlon, an official may arbitrarily designate which player will begin the game.

B. All Pattern Blocks are spread out to the side of the gameboard so that they are readily available to both players. This area is referred to as the bank.

**RULES:**

1. **Alternating Turns and Selection of Pieces:** Players alternate taking turns. On a player's turn, one, two or three different colored Pattern Block playing pieces are selected to be placed onto the gameboard. A player *may not* select two or three pieces which are the same color.

2. **Selection before Placement:** The choice of whether a player selects one, two, or three different colored Pattern

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Blocks is entirely up to the player. But players must pull out all desired pieces from the bank and hold them in their possession before placing the selected pieces onto the gameboard. Once a player places a piece onto the gameboard, no other pieces can be selected from the bank.

3. **Pattern Block Placement:** Each player in turn places all selected pieces onto the gameboard. Pieces can be placed anywhere on the gameboard's designated regions provided each piece placed either completely covers a geometric region on the gameboard or allows space for the geometric figure to be completed by Pattern Block pieces. For example, a triangle can be placed onto one of the gameboard's hexagon regions since a trapezoid and blue rhombus can be played at some later time to complete the figure. On the other hand, a tan rhombus or square *can not* be placed into a hexagon region since such positioning does not allow the region to be completely covered by other Pattern Block pieces.
4. **Placement within Regions:** Pattern Block pieces may not be placed across a solid line. All playing pieces must be placed within one of the outlined regions (shapes) on the gameboard.
5. **Placed Pieces *may not* be moved:** Once a piece has been placed onto the gameboard, it may not be moved to another location (shape) nor to a different part of the same shape (region).
6. **Selected Pieces Must Be Placed:** Once a piece has been selected, it must be placed during the player's turn. If a player is unable to place all of the selected pieces onto the gameboard, the opposing player will be declared the winner. Even if a player has inadvertently violated rule 1 by selecting more than one piece of the same color, all pieces must be placed onto the gameboard. The opposing player will have an opportunity to challenge the inappropriate selection before starting a turn.
7. **A Piece Laid Is A Piece Played:** Once a player's hand is removed from a piece placed onto one of the gameboard's region, it may not be moved to another location.
8. **Beginning of a Turn:** A player's turn begins when an opponent's hand is removed from the last selected piece placed onto the gameboard.
9. **End of a Turn:** A player's turn ends when the player's hand is removed from the last selected piece as it is placed onto the gameboard.


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10. **Winning:** A player may win because of an opponent's mistake in drawing pieces that cannot be placed onto the gameboard (rule 6) or by being the last player able to place a piece or pieces onto the unoccupied spaces on the gameboard.

**TOURNAMENT RULES:**

11. **Challenges:** If a player violates any of the preceding rules, the opposing player may issue a challenge after a player's hand is removed from placing the last selected piece. Challenges *may not* be issued after selecting Pattern Blocks from the bank. Should a player incorrectly select pieces to be placed onto the gameboard, the opponent may challenge the play. A Pentathlon official will rule on any challenge. If the challenge is ruled to be correct, the challenger can remove all or some of the pieces which the challenged player placed onto the gameboard. All Pattern Blocks violating rules 3 and 4 must be removed if a player is challenged for violating these rules. If a challenge is ruled to be incorrect, the challenger must forfeit a turn.
12. **One Minute Time Limit:** Each player must complete a turn within one minute. If a player fails to complete a turn in one minute the next turn is forfeited. In this instance, an opposing player would be allowed to complete two successive turns. The third violation of this rule results in a forfeiture of the game. Players may not challenge each other over this rule at the Pentathlon. An official at the tournament will decide if this rule is being violated.
13. **Game Time Limit:** At the Pentathlon, if the fifteen (15) minutes of game time has run out before either play has ended the game, a tie will be declared. If a player is in the process of completing a turn when game time is called, that turn will be allowed to be completed.

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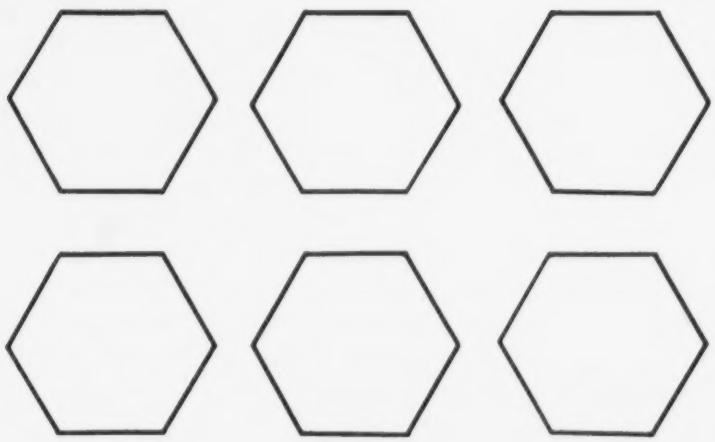
# GAMEPIECES FOR HEX!

## DUPLICATE THIS PAGE

Please color these YELLOW.



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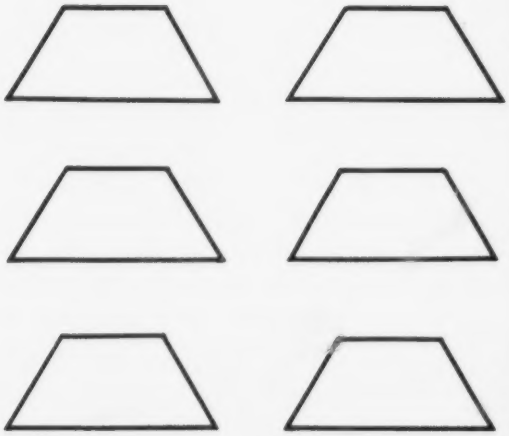

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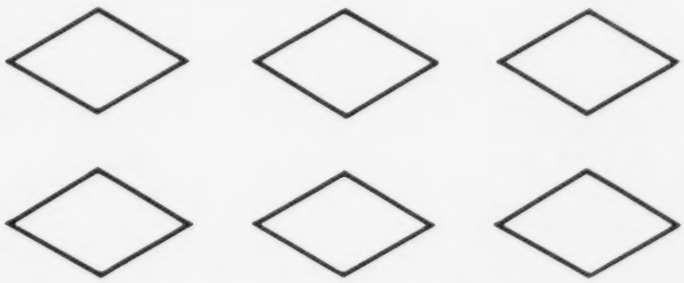

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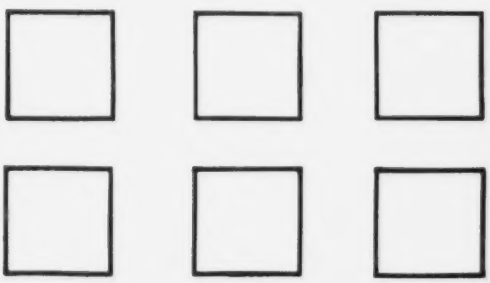



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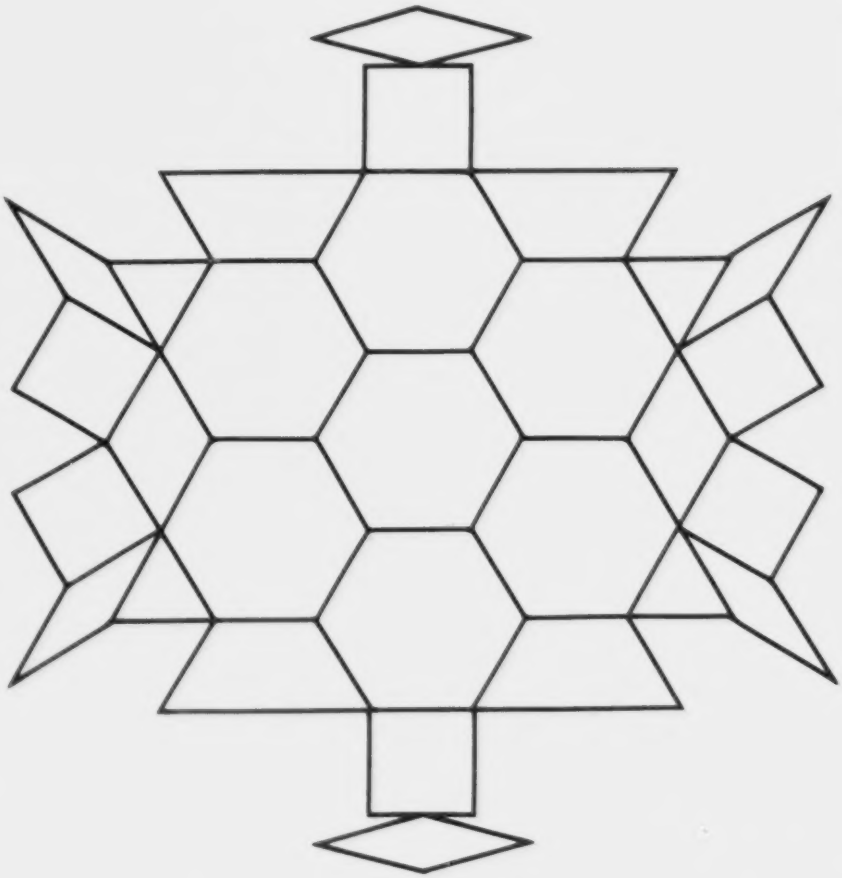
# HEX!



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# SHAPE-UP

**Created by**
**Dr. John C. del Regato**

This Mathematics Pentathlon™ event uses a specially constructed gameboard to practice shape and size recognition. Although the underlying addition principle used in this game is easily mastered by adults, many younger learners involved in this Pentathlon game will initially have some difficulty in coordinating their moves to successfully master this aspect of the game. Teachers and parents should not be overly concerned about this since it is a natural developmental characteristic. The alternating light and dark shapes as well as the various positionings of the different shapes have been purposely designed to complicate the task of matching shapes that differ by size (coloring and positioning being unimportant). Before playing this game a teacher or parent may wish to introduce a young learner to the various shapes found on the gameboard (circle, square, rectangle, triangle, pentagon, ellipse, and star). Young players of this game should also be given practice in placing chips of the same color onto a large and small version of the same shape so that the goal of the game is understood.

**UNDERLYING MATHEMATICS/SCIENCE:** counting, addition (the family of facts), shape recognition, classification, observation skills, combinatorics

**MATERIALS:** Official Mathematics Pentathlon™ gameboard, one regular die, four transparent chips — two of one color and two of another

**TIME:** Total tournament playing time is 15 minutes. Each player has one minute to complete a turn.

**GOAL:** To win, a player must have both markers on spaces containing the same shape. One space must contain a large version and the other a small version of the same shape. Size is the important attribute difference, not color or position of shape. A win is declared when this situation occurs and the player says "Shape Up."

**TO START:**

A. Players alternate in throwing the die to determine who begins the game. Lowest roll of the die begins.

B. The beginning player selects two transparent markers (chips) of the same color. The remaining two markers belong to the opponent.

C. Each player places these markers into the start box so designated on the gameboard.

**RULES:**

1. **Alternating Turns and Movement:** Players alternate in taking turns by rolling the die and moving their marker(s) the number of spaces indicated by the die. The die indicates the total number of spaces that can be moved.

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For example, if the die shows 3, a player can leave one marker where it is and move the other marker three spaces in the chosen direction. Another option would be to move one marker one space and the other marker two spaces for a total of three spaces moved.

2. **Moving a Touched Marker:** Once a marker has been touched by a player during a turn, it must be moved as a part of that turn.
3. **Direction of Movement:** Markers enter from the start box onto one of the two circular spaces touched by the start box. A marker can be moved either clock-wise or counter clock-wise in direction along the path of circular spaces containing various shapes. A player *may* choose to move one marker in one direction and the other marker in the opposite direction during a turn. The total number of spaces moved is determined by the die.
4. **No Back-Tracking:** During a turn, a player may not change the direction of a marker's movement. For example, in order to complete a five-space move, a player *may not* move a marker three spaces in a clock-wise direction and then reverse the direction of movement for the same marker and move back two spaces.
5. **Bumping a Player's Marker:** During a turn if a player chooses to place a marker onto a space occupied by an opponent's marker or onto a space containing a figure of the same shape and size as one on which an opponent's marker is placed, the opponent's marker is returned to the start box. It is entirely possible that both of the opponent's markers can be sent back to the start box by exercising this rule. The player making use of this rule is to move the opponent's markers into the start box.
6. **Beginning of a Turn:** A player's turn begins when the die is rolled.
7. **End of Turn:** A player's turn ends when the die is released into the hand of the opposing player.
8. **Win by Announcing "Shape-Up":** A winning position has been attained when both of a player's markers are on spaces containing the same shape but with one being the large version and other being the small version of the same shape (color and positioning of the shapes are not important). The player must also recognize that this is a winning position by announcing "Shape-Up" and thus declaring a win. If the die is passed to the opponent and the winning position is not announced, play con-


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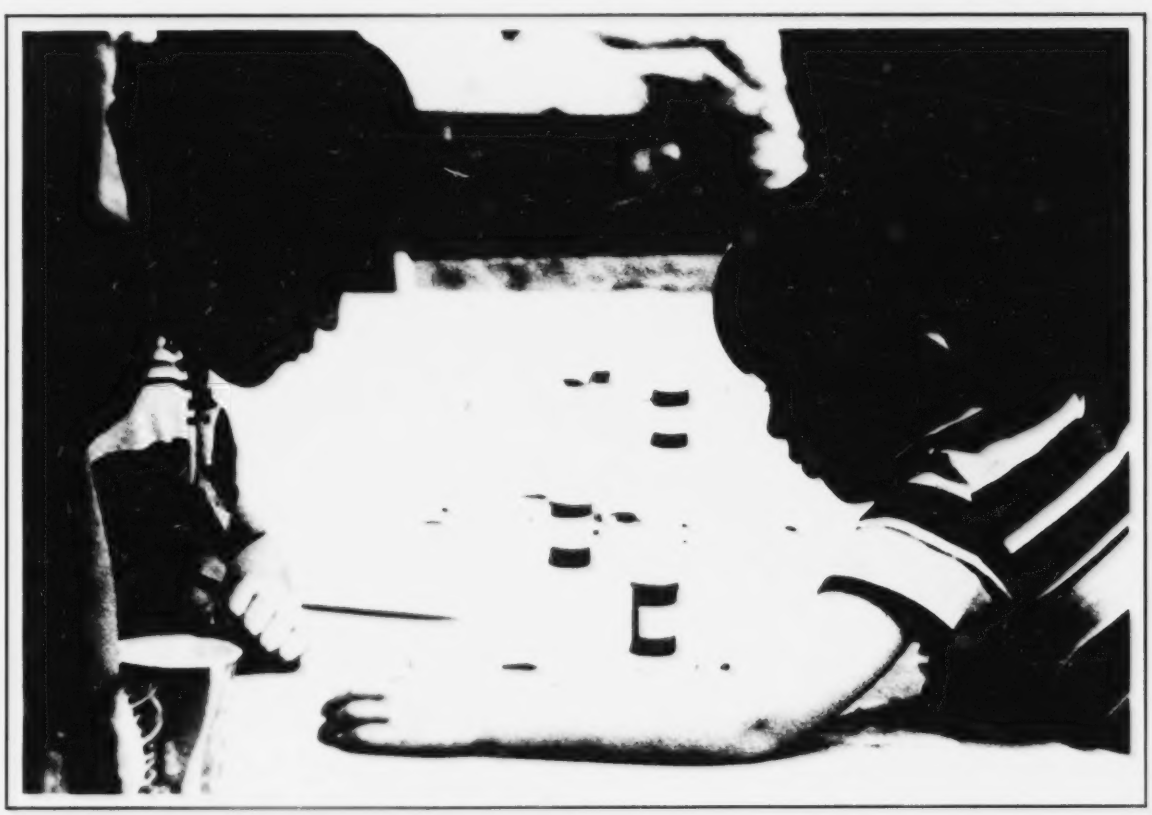
tinues. When a player has received the die and a winning position has been constructed by the opponent, the player may announce "Shape Up" and be declared the winner even though the winning position was made by the opponent's markers. This must be done prior to rolling the die. Once a player has rolled the die a player may not declare a win on an opponent's winning position.

9. **Reconstruction of a Winning Position**: If both players failed to declare the winning position, the player who constructed the winning position may not win by declaring it at the beginning of the next turn. Instead, the player must roll the die again and attempt to reconstruct a winning position.
10. **Starting Box:** With the exception of rule 5 and the time limit penalty and challenge penalty given below, markers may not be moved into or through the starting box once the game has begun.



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**TOURNAMENT RULES:**

11. **Challenges:** If a player violates any of the preceding rules, the opposing player may issue a challenge upon receiving the die. Challenges *may not* be issued after the die has been rolled. Should a player incorrectly count the number of spaces to be moved or if they incorrectly split up the amount shown on the die, the opponent should challenge the play. A Pentathlon official will rule on any challenge. If the challenge is ruled to be correct, the challenged player moves both markers back to the start box and the challenger begins a turn. If the challenge is ruled to be incorrect, both the challenger's markers are placed into the start box and the challenger loses a turn.
12. **One Minute Time Limit:** Each player must complete a turn within one minute. If a player fails to complete a turn in one minute both markers must be returned to the start box and the die given to the opponent. The third violation of this rule results in a forfeiture of the game. Players may not challenge each other over this rule at the Pentathlon. An official at the tournament will decide if this rule is being violated.
13. **Game Time Limit:** At the Pentathlon, if the fifteen (15) minutes of game time has run out before either play has ended the game, a tie will be declared. If a player is in the process of completing a turn when game time is called, that turn will be allowed to be completed.


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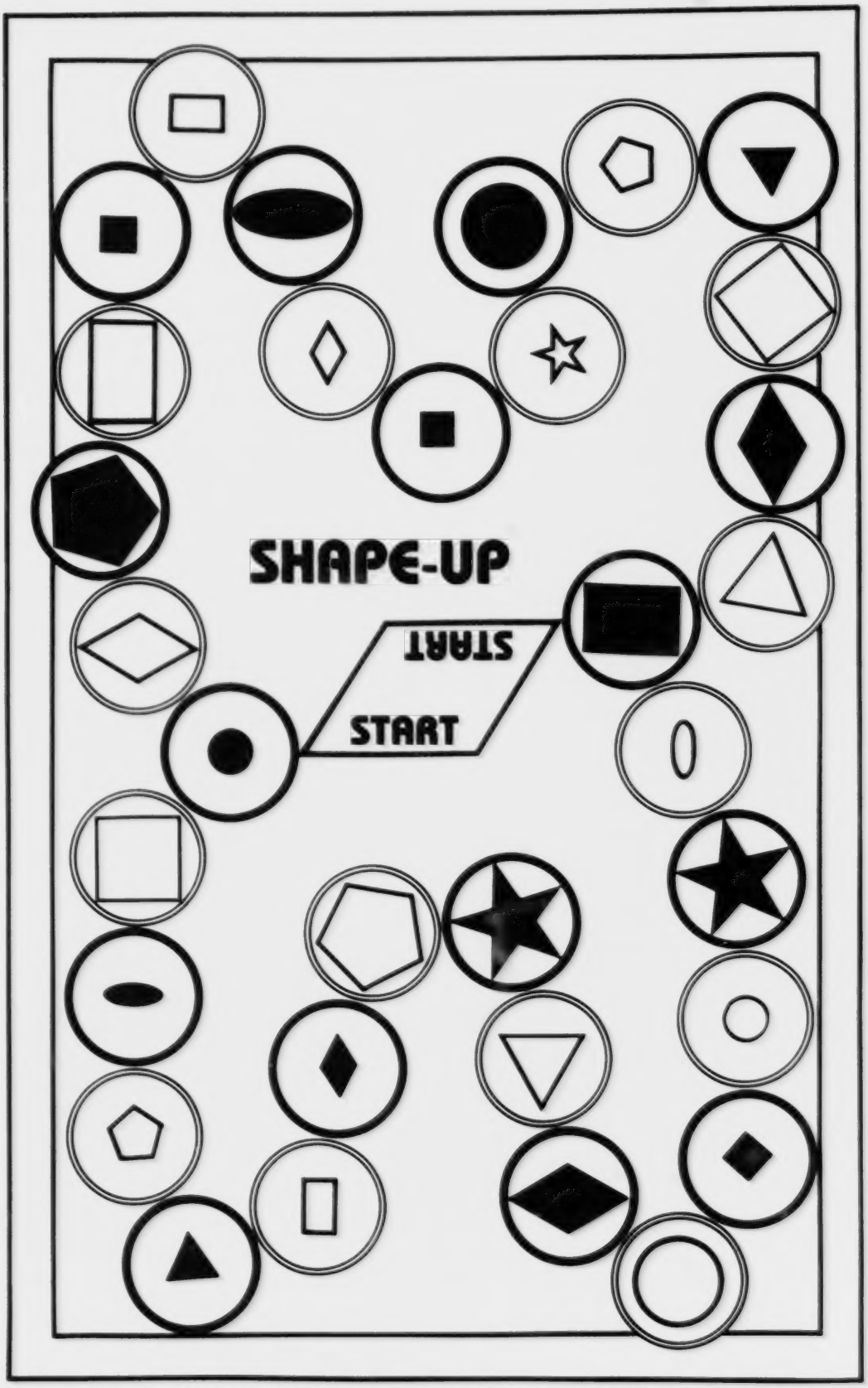


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# STAR TRACK

**Created by**
**Dr. John C. del Regato**

This race game presents two special features: pentagrams (stars) and numerals. Both of these features lend excitement for children as they practice counting and numeral recognition. In introducing the game to children, a teacher or parent may wish to concentrate only on the rules pertaining to the stars. As children gain familiarity with the game, the rules pertaining to the numerals can then be introduced. Participants at the tournament will play by the following rules.

**UNDERLYING MATHEMATICS/SCIENCE:** counting, numeral recognition, number-numeral association

**MATERIALS:** Official Mathematics Pentathlon™ gameboard; two pawns of differing color; links, beads, or paperclips connected in the following manner:

2 chains of 9 links
3 chains of 8 links
4 chains of 7 links
5 chains of 6 links
6 chains of 5 links
7 chains of 4 links
8 chains of 3 links
9 chains of 2 links

**TIME:** Total tournament playing time is 15 minutes. Each player has one minute to complete a turn.

**GOAL:** To win, a player must reach the last pentagon when a round has been completed. This can be accomplished by drawing a chain with a number of links that is greater than or equal to the number necessary to land on the final pentagon. If both players succeed in reaching the final pentagon during a round, a tie is declared.

**TO START:**

A. Each player places his or her game pawn on one of the two starting pentagon spaces.

B. All chains of differing lengths are placed into a container that will not allow either player to view the length of the chain being selected.

C. Each player selects a chain from the container. The chains are compared. The player with the longest chain begins by moving the appropriate pawn forward as many pentagon spaces as there are links in the chain. The

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second player is handed the beginning player's chain and if no challenge is made, both the beginning player's chain and the second player's chain are placed into a discard pile.

D. In beginning the game, if both players choose the same length chain, both chains are placed into a discard pile and each player chooses another chain. This process is continued until one player has picked a chain that varies in length from the other player.

**RULES:**

1. **Alternating Turns:** Players alternate turns in drawing one chain from the container, counting the links and moving that number of spaces forward on the gameboard. Chains are handed to the opposing player and if no challenge is made, the chain is placed into the discard pile.
2. **Starred Spaces:** If a pawn lands on a starred space, it must be replaced to the previous starred position. If a pawn lands on the first starred space, it must be returned to the START position.
3. **Numbered Spaces:** If a pawn lands on a numbered space, it is advanced as many pentagon spaces as the number indicates.
4. **End of Turn:** A turn ends when a player has released the chain to the opposing player.
5. **Beginning of a Turn:** A player's turn begins either:
   a. at the signaled beginning of the game,
   b. when a player has placed (released hold of) the opponent's chain into the discard pile, or
   c. when an official has indicated play to continue after a challenge has been issued (see challenge rules below).
6. **Selected Chain Must Be Used:** A player must take the first chain of links touched. A player may not observe the length of a chain and then attempt to make a different selection.
7. **Landing on an Occupied Space:** Should a player's pawn land on a space occupied by the opponent's pawn, the opponent's marker is moved back two spaces. Should this cause the opponent's marker to land on a starred or numbered space, rules 2 and 3 are ignored.
8. **Reuse of Chains:** If all chains have been placed into the discard pile and neither player has won nor has there been a declared tie, the discard pile of chains is placed back into the container and play continues.



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9. **Win/Tie:** A player does not need to get the exact number to reach the final pentagon space. A chain containing the number of links greater than or equal to the number of spaces needed to reach the final pentagon has allowed a player to win. If after both the beginning player and second player (a round) have completed their respective turns, and one player lands on the last pentagon space, a win is declared for that player. If both players succeed in doing this at the end of a round, a tie is announced.

**TOURNAMENT RULES:**

10. **Challenges:** If a player violates any of the preceding rules, the opposing player may issue a challenge. Should a player incorrectly count the number of links in the chain, and be challenged by an opponent, the tournament official will decide if the challenge is correct. For any challenge, if the challenger is correct the opposing player's pawn must be moved back two starred positions and the player must forfeit that turn. If the challenger is incorrect, the challenger's pawn is moved back two starred positions and the challenger loses a turn.
11. **One Minute Time Limit:** Players must complete their turn within one minute. Failure to do so will result in a player's pawn being moved back to the previous starred space. The third violation of this time constraint will result in a forfeiture of the game. Players may not challenge each other over this rule at the Pentathlon. An official at the tournament will decide if this rule is being violated.
12. **Game Time Limit:** At the Pentathlon, if the fifteen (15) minutes of game time has run out before either player has ended the game, a tie will be declared. If a round has not been completed when game time is called, that round will be allowed to be completed.

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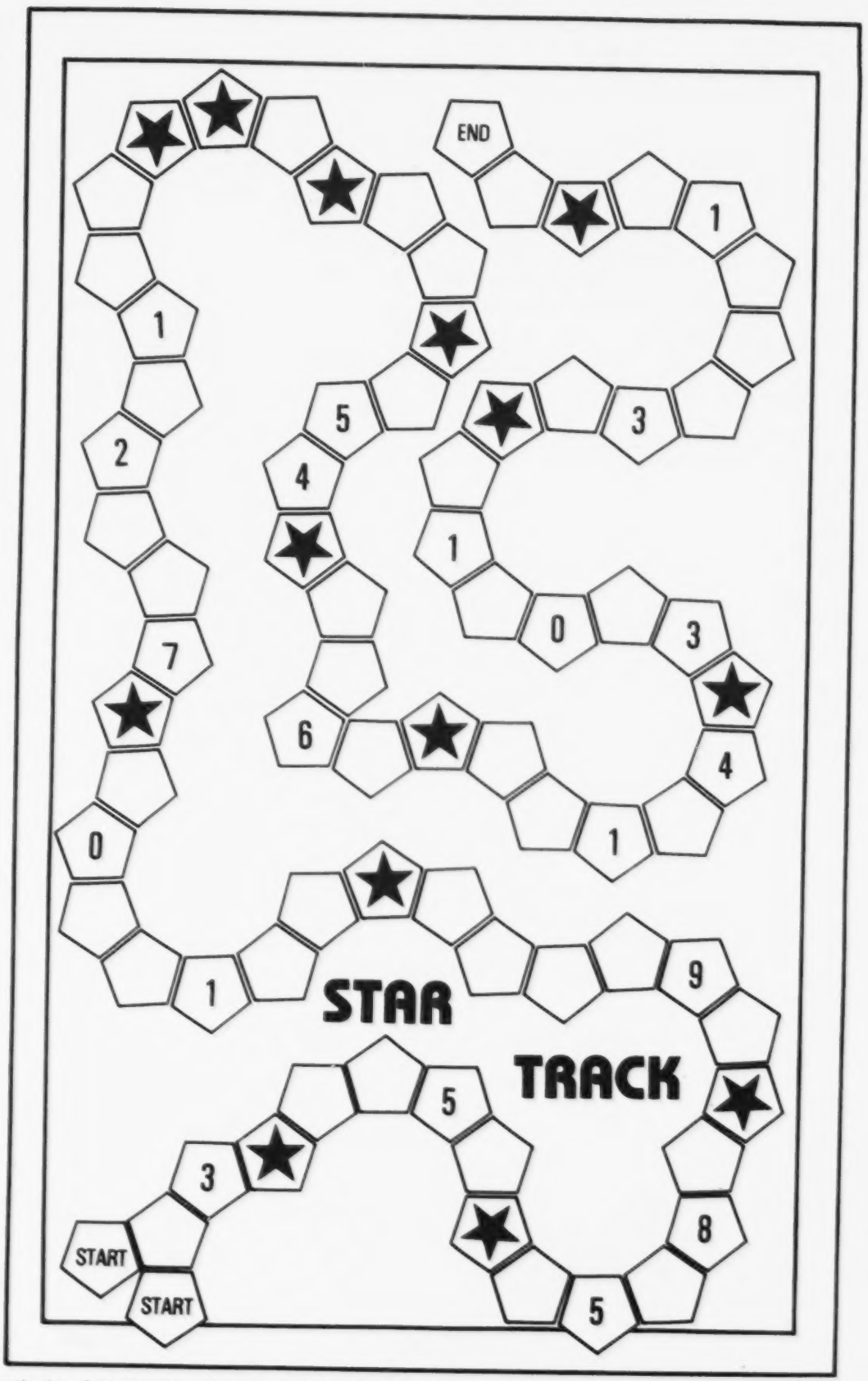


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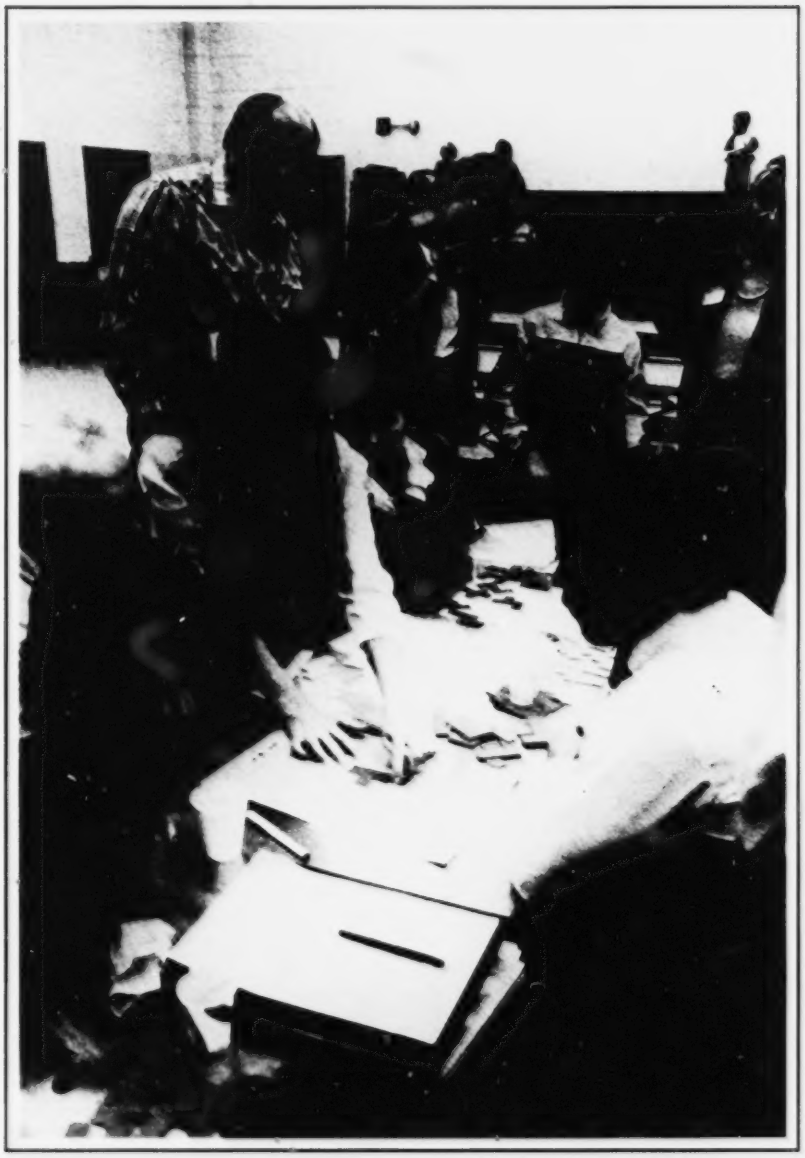

Orientation of parents, teachers, and administrators.



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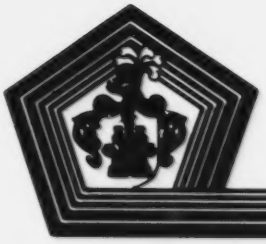

# NOTES

**Paula Minton**
**One of our star volunteers.**

The Pentathlon Institute offers Mathematics Pentathlon® orientation sessions or special in-service sessions on teaching Mathematics Experienced-Based Approach (MEBA™). For further information please contact:

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President, Pentathlon Institute

Mary Gilfeather
National Director, Mathematics Pentathlon®

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